Consequence-Driven Reasoning for Horn-SHIQ Ontologies

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Outline

Results Overview

- Classification times for some well-known large ontologies:

	GO	NCI	Galen v.0	Galen v.7	SNOMED
Concepts:	20465	27652	2748	23136	389472
FaCT++	15.24	6.05	465.35	—	650.37
HermiT	199.52	169.47	45.72	—	—
Pellet	72.02	26.47	—	—	—
CEL	1.84	5.76	—	—	1185.70

RESULTS OVERVIEW

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CB	1.17	3.57	0.32	9.58	49.44
Speed-Up:	1.57X	1.61X	143X	∞	13.15X

- The improvement is obtained using a new consequence-based reasoning procedure

available at: `cb-reasoner.googlecode.com`

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Ontology

$$\mathrm{H} \equiv \mathrm{MO} \sqcap \exists \mathrm{isPartOf}.\mathrm{CS}$$
$$\mathrm{MO} \equiv \mathrm{O} \sqcap \exists \mathrm{isPartOf}.\mathrm{MS}$$
$$\mathrm{MS} \sqsubseteq \mathrm{BS}$$
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$$?- \mathrm{MS} \sqsubseteq \mathrm{BS}$$

Taxonomy

$\top$

BS O

CS MS MO

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?− $\mathsf{MS} \sqsubseteq \mathsf{BS}$ *Yes*

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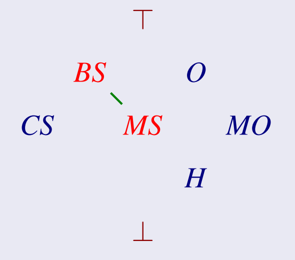

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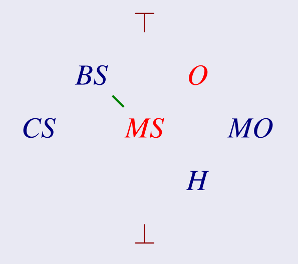

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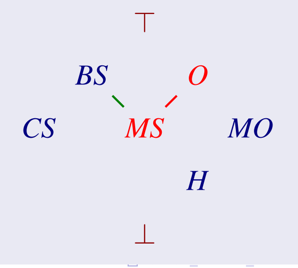

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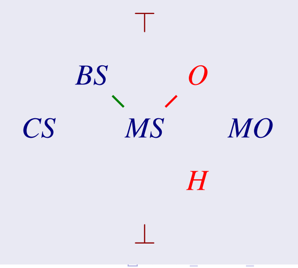

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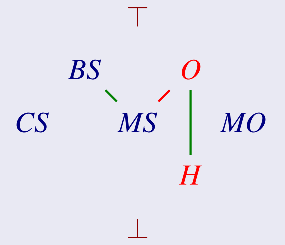

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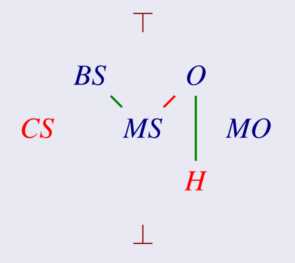

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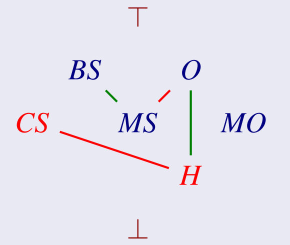

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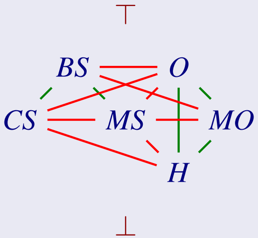

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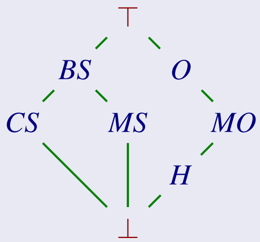

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3. Large and highly cyclic models caused by existential dependencies $C \sqsubseteq \exists R.D$ (especially apparent for Galen)

ONTOLOGY

$$\begin{aligned} \text{Heart} &\sqsubseteq \exists \text{isPartOf}.\text{CirculatorySystem} \\ \text{CirculatorySystem} &\sqsubseteq \exists \text{hasPart}.\text{LeftLung} \\ \text{LeftLung} &\sqsubseteq \exists \text{isPartOf}.\text{RespiratorySystem} \\ \text{RespiratorySystem} &\sqsubseteq \exists \text{hasPart}.\text{Trachea} \end{aligned}$$

and so on . . .

Outline

$\mathcal{EL}$ Family of DLs

- $\mathcal{EL}$ [Baader et al.,IJCAI 2003,2005] is a lightweight DL:
  - concepts are constructing using $\top$, $C \sqcap D$, and $\exists R.C$
  - axioms are $C \sqsubseteq D$ and $C \equiv D$
  - $\mathcal{EL}^{++}$ adds $\bot$, $R_1 \cdots R_n \sqsubseteq R$, nominals, safe datatypes

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- Most of the axioms in Galen are expressed in $\mathcal{ELH}$

Example

KidneyExamination $\equiv$ ClinicalAct $\sqcap$
$\exists$hasSubprocess.(ExaminingProcess $\sqcap$ $\exists$involves.Kidney)

$\mathcal{EL}$ Classification Procedure

1. Normalization to simple axioms of the forms:
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CR1 $\dfrac{A \sqsubseteq B}{A \sqsubseteq C} : B \sqsubseteq C \in \mathcal{O}$ CR2 $\dfrac{A \sqsubseteq B \quad A \sqsubseteq C}{A \sqsubseteq D} : B \sqcap C \sqsubseteq D \in \mathcal{O}$

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CR5 $\dfrac{A \sqsubseteq \exists r.B \quad B \sqsubseteq C}{A \sqsubseteq D} : \exists r.C \sqsubseteq D \in \mathcal{O}$ (language = $\mathcal{ELH}$)

ADVANTAGES

- Performs the full classification "in one pass"
- Derives only subsumptions that are implied ($< 1\%$ of all)
- No non-determinism, no backtracking
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- Easy to make incremental / parallel / distributed, add explanations, progress bar…

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The "only" disadvantage:

- the language may be too restricted
  (tractable reasoning is the primary focus)

GALEN

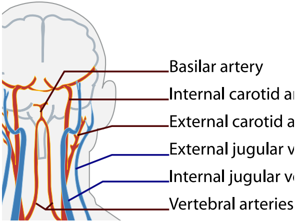
- Galen uses two constructors that are outside of $\mathcal{EL}^{++}$: inverse roles and role functionality:

EXAMPLE (GALEN)

$$\text{BasilarArtery} \sqsubseteq \exists \text{isBranchOf}.\text{VertebalArtery}$$
$$\text{VertebalArtery} \sqsubseteq \exists \text{hasBranch}.\text{BasilarArtery}$$

➤ $\text{isBranchOf} \equiv \text{hasBranch}^{-}$

➤ $\mathit{Fun}(\text{isBranchOf})$



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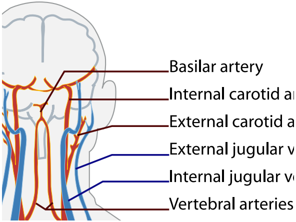

- Adding any of these constructors to $\mathcal{ELH}$ results in complexity increase from PTime to ExpTime [Baader, Brandt, Lutz 2005; 2008]

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- Galen uses two constructors that are outside of $\mathcal{EL}^{++}$: inverse roles and role functionality:

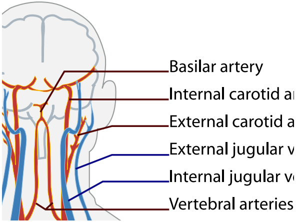
Example (Galen)

$$\text{BasilarArtery} \sqsubseteq \exists \text{isBranchOf}.\text{VertebalArtery}$$

$$\text{VertebalArtery} \sqsubseteq \exists \text{hasBranch}.\text{BasilarArtery}$$

➤ $\text{isBranchOf} \equiv \text{hasBranch}^{-}$

➤ $Fun(\text{isBranchOf})$



- Adding any of these constructors to $\mathcal{ELH}$ results in complexity increase from PTime to ExpTime [Baader, Brandt, Lutz 2005; 2008]
- We are not scared of the high complexity!

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Example

- $A \sqsubseteq \forall R.(\neg B)$ $\quad$ $\exists R^-.A \sqsubseteq \leqslant 1\, R.(B \sqcup C)$ $\quad$ are OK
- $A \sqsubseteq B \sqcup C$ $\quad$ $\forall R.A \sqsubseteq B$ $\quad$ $A \sqsubseteq \leqslant 2\, R.B$ $\quad$ are not OK

NEW INFERENCE RULES

- Interactions between existential and universal restrictions:

1 $\dfrac{A \sqsubseteq \exists R.B \quad B \sqsubseteq \forall R^{-}.C}{A \sqsubseteq C}$ $\leftrightsquigarrow$ **CR5** $\dfrac{A \sqsubseteq \exists R.B \quad \exists R.B \sqsubseteq C}{A \sqsubseteq C}$

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all rules

- The general form of derived axioms:

$$\bigsqcap A_i \sqsubseteq B \qquad \bigsqcap A_i \sqsubseteq \exists R.\bigsqcap B_j$$

– exponentially-many in worst case (which is optimal)

RESULTS

- A novel classification procedure for Horn $\mathcal{SHIQ}$ ontologies
- Key advantages over model-building procedures:
  1. deterministic
  2. full classification in one pass
  3. no problems with existential dependencies $C \sqsubseteq \exists R.D$
  4. optimal for Horn-$\mathcal{SHIQ}$ and $\mathcal{ELH}$ (pay-as-you-go).
- The implementation exhibits a significant speedup:

	GO	NCI	Galen v.0	Galen v.7	SNOMED
FACT++	15.24	6.05	465.35	—	650.37
HERMIT	199.52	169.47	45.72	—	—
PELLET	72.02	26.47	—	—	—
CEL	1.84	5.76	—	—	1185.70
CB	1.17	3.57	0.32	9.58	49.44
Speed-Up:	1.57X	1.61X	143X	∞	13.15X

available at: `cb-reasoner.googlecode.com`

The Inference Rules for Horn $\mathcal{SHIQ}$

$$\frac{}{M \sqcap A \sqsubseteq A} \qquad \frac{}{M \sqsubseteq \top} \qquad \frac{M \sqsubseteq A_1 \quad \dots \quad M \sqsubseteq A_n}{M \sqsubseteq C} : \bigsqcap_{i=1}^{n} A_i \sqsubseteq C \in \mathcal{O}$$

$$\frac{M \sqsubseteq \exists R.N \quad N \sqsubseteq \bot}{M \sqsubseteq \bot} \qquad \frac{M \sqsubseteq \exists R_1.N \quad M \sqsubseteq \forall R_2.A}{M \sqsubseteq \exists R_1.(N \sqcap A)} : R_1 \sqsubseteq_{\mathcal{O}} R_2$$

$$\frac{M \sqsubseteq \exists R_1.N \quad N \sqsubseteq \forall R_2.A}{M \sqsubseteq A} : R_1 \sqsubseteq_{\mathcal{O}} R_2^-$$

$$\frac{\begin{array}{ll} M \sqsubseteq \exists R_1.N_1 & N_1 \sqsubseteq B \\ M \sqsubseteq \exists R_2.N_2 & N_2 \sqsubseteq B \\ M \sqsubseteq \leqslant 1\, S.B & \end{array}}{M \sqsubseteq \exists R_1.(N_1 \sqcap N_2)} : \begin{array}{l} R_1 \sqsubseteq_{\mathcal{O}} S \\ R_2 \sqsubseteq_{\mathcal{O}} S \end{array}$$

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Where $M, N = \bigsqcap A_i$