Role Conjunctions in Expressive Description Logics

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Outline

Summary of the Main Results

Known Results (see DL Complexity Navigator[1])

(Finite model) reasoning is:

- NExpTime-complete for $\mathcal{SHOIQ}$ [**OWL**]
- ExpTime-complete for $\mathcal{SHQ}$ and $\mathcal{SHIQ}$ [**OWL**-Lite]

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- 2ExpTime-complete for $\mathcal{SHIQ}^{\sqcap}$ [hard already for $\mathcal{SHI}^{\sqcap}$]
- ExpTime-complete for $\mathcal{SHQ}^{\sqcap}$

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- ExpTime-complete for $\mathcal{SHQ}^{\sqcap}$

The exponential blowup is due to a combination of:
role conjunctions + inverses + role inclusions + transitive roles

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Motivation I: Role Constructors in OWL

- OWL (= $\mathcal{SHOIQ}$) has a rich algebra of concept constructors:

Conjunction	$C \sqcap D$	Mammal $\sqcap$ Predator
Disjunction	$C \sqcup D$	Male $\sqcup$ Female
Negation	$\neg C$	$\neg$Vegetarian
Existential Restriction	$\exists R.C$	$\exists$produce.Oxygen
Universal Restriction	$\forall R.C$	$\forall$eat.Plant
Number Restrictions	$\geqslant n\, R.C$	$\geqslant 8$ hasPart.Leg

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- The dis-balance is compensated by concept / role axioms:

Concept Inclusion	$C \sqsubseteq D$	$\forall$eat.Plant $\sqsubseteq$ $\neg$Predator
Role Inclusion	$R \sqsubseteq S$	eat $\sqsubseteq$ consume
Assertions	$\langle a, b\rangle : R$	$\langle Bill, John\rangle$: hasFather
Transitivity	$Tra(R)$	Tra(hasDescendant)
Functionality	$Fun(R)$	Fun(hasFather)

Motivation I: Role Constructors in OWL

OWL 2 directions: new role axioms

- New role assertions:

Symmetry	$Sym(R)$	$Sym(\text{hasBrother})$
Anti-Symmetry	$Asy(R)$	$Asy(\text{hasParent})$
Reflexivity	$Ref(R)$	$Ref(\text{knows})$
Irreflexivity	$Irr(R)$	$Irr(\text{hasChild})$
Disjointness	$Disj(R, S)$	$Disj(\text{hasParent}, \text{hasUncle})$

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- Complex role inclusion axioms:

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What about role constructors?

- The simplest one is:

Role Conjunction $R \sqcap S$ | $\text{Man} \sqcap \exists(\text{cooks} \sqcap \text{eats}).\text{Soup}$

Motivation II: Conjunctive Queries

- Answering conjunctive queries w.r.t. knowledge bases

$$Q(x) = \langle x \rangle \longleftarrow \text{Man}(x) \wedge \text{cooks}(x, y) \wedge \text{eats}(x, y) \wedge \text{Soup}(y)$$

  - given: **TBox**, **ABox**, $Q(x)$
  - find: $\langle x \rangle$ such that **TBox**, **ABox** $\models Q(x)$

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- It is known that conjunctive query answering in $\mathcal{SHIQ}$ can be reduced to standard reasoning in $\mathcal{SHIQ}^{\sqcap}$.
- Reasoning in $\mathcal{SHIQ}^{\sqcap}$ can be done 2ExpTime, whereas $\mathcal{SHIQ}$ is merely ExpTime.
- It was not clear whether this bound is tight.

Outline

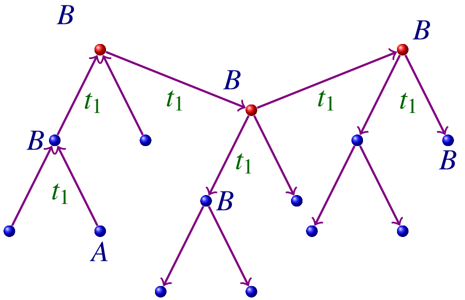
The Exponential Blowup in $\mathcal{SHIQ}^{\sqcap}$

TBox

$A \sqsubseteq \forall r_1.B$

$t_1 \sqsubseteq r_1$

$Tra(t_1)$



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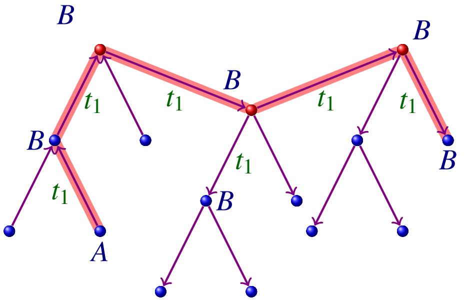

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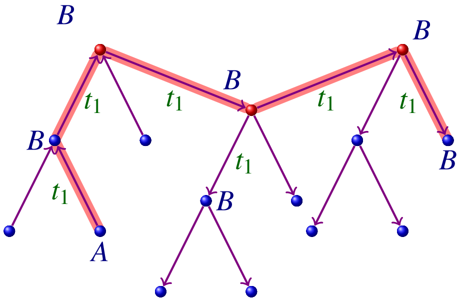

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  - works even without tree-model property (e.g. for $\mathcal{SHOIQ}$)

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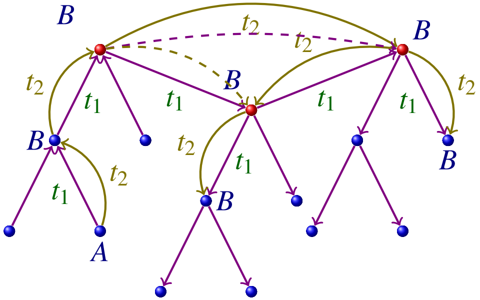

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- Similar technique works for $\mathcal{SHIQ}^{\sqcap}$, except that
  - tree-model property is crucial (does not work for $\mathcal{SHOIQ}$)
  - can produce exponentially-many axioms—one for every combination of transitive subroles

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$A \sqsubseteq \forall(t_1 \sqcap t_2).A^{(t_1 \sqcap t_2)}$

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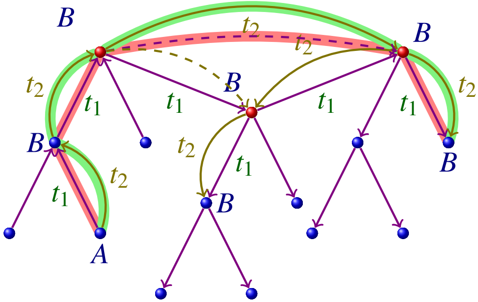

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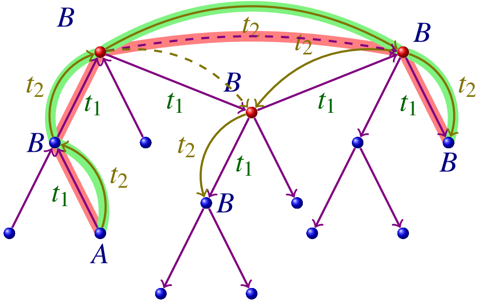

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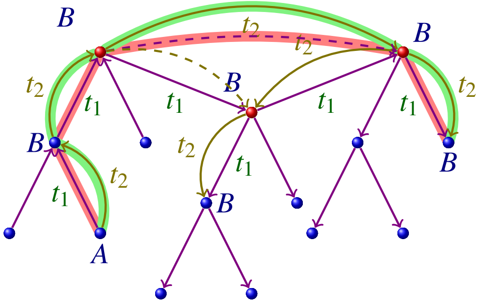

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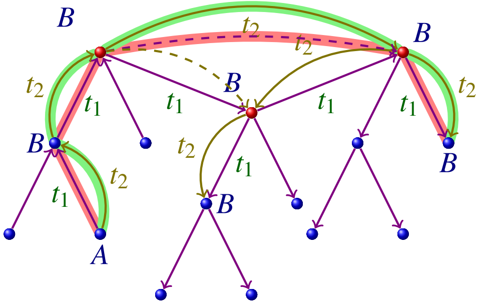

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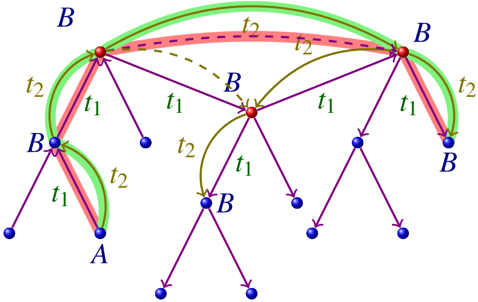

- The exponential blowup does not take place when either:
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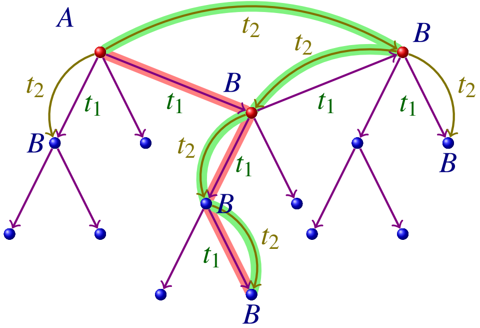

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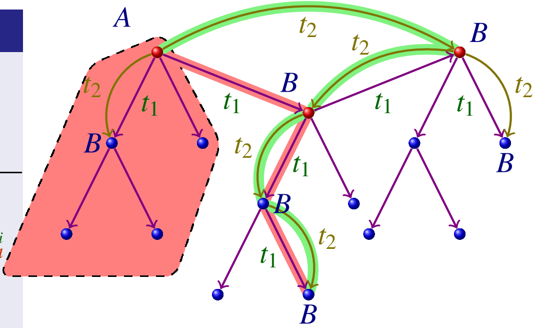

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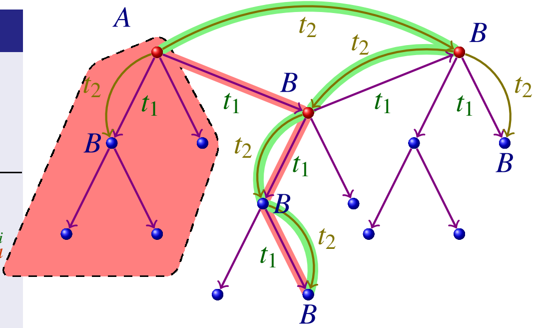

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  - from a root element, or
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- The main idea: remember from which tree an element is reachable by tagging concepts with individuals
- This translation is polynomial, hence $\mathcal{SHQ}^{\sqcap}$ is in ExpTime

Outline

WHY IS $\mathcal{SHIQ}^{\sqcap}$ HARDER?

TBox

$s_1 \sqsubseteq t_2$	$s_1 \sqsubseteq t_3$	$s_1 \sqsubseteq t_4$
$s_2 \sqsubseteq t_1$	$s_2 \sqsubseteq t_3$	$s_2 \sqsubseteq t_4$
$s_3 \sqsubseteq t_1$	$s_3 \sqsubseteq t_2$	$s_3 \sqsubseteq t_4$
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$t_1 \sqsubseteq r_1$	$t_2 \sqsubseteq r_1$
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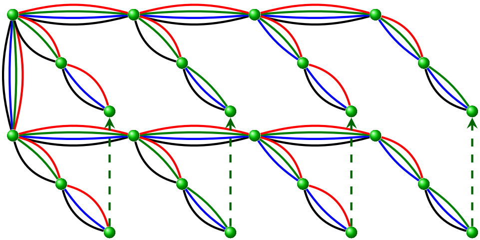
$\rho := (r_1 \sqcap r_2)$



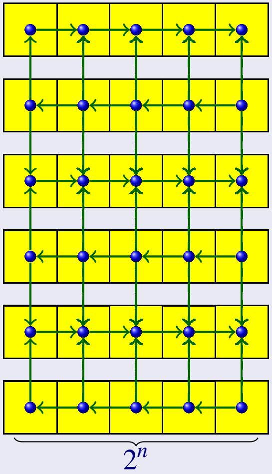
- Using role conjunctions it is possible to connect the corresponding elements in exponentially-long chains

THE HARDNESS RESULT FOR $\mathcal{SHIQ}^{\sqcap}$

By reduction from the word problem for an exponential-space alternating Turing machine:

- Configurations are encoded on exponential chains

COMPUTATION OF TM



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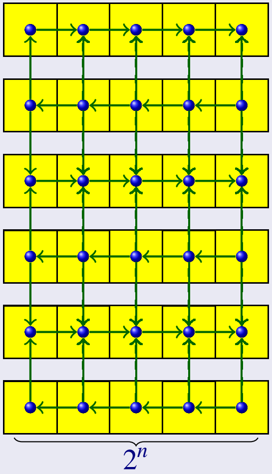

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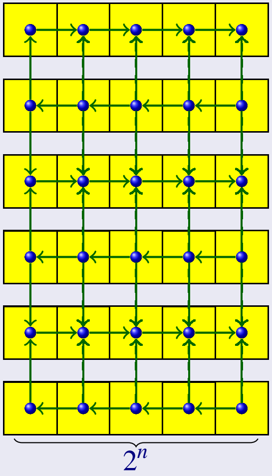

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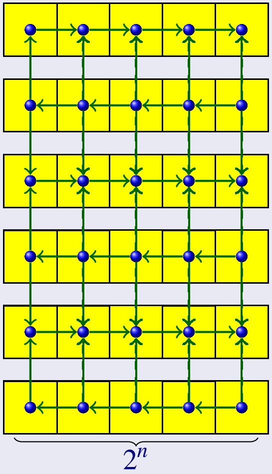
- Corresponding cells of successive configurations are connected by $\rho = R_1 \sqcap \cdots \sqcap R_n$
- Easy to simulate the computation
- Since $\text{AExpSpace} = \text{2ExpTime}$ we have:

Theorem

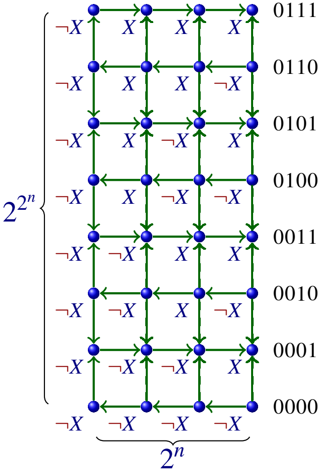
(Finite model) reasoning in $\mathcal{SHI}^{\sqcap}$ (and therefore in $\mathcal{SHIQ}^{\sqcap}$) is 2ExpTime-hard.

Computation of TM



DOUBLY-EXPONENTIAL CHAINS IN $\mathcal{SHIQ}^{\sqcap}$

- Encode the counter on exponentially-long chains
  - the value of X on i-th element of the chain encodes the i-th bit



DOUBLY-EXPONENTIAL CHAINS IN $\mathcal{SHIQ}^{\sqcap}$

- Encode the counter on exponentially-long chains
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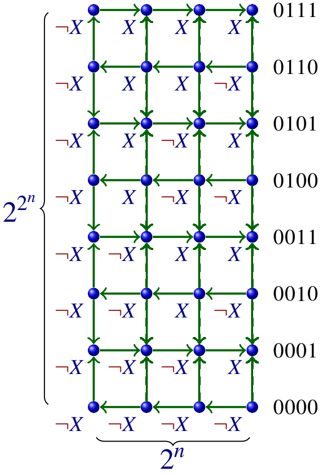
- Incrementing of the counter:
  - the least bit is always flipped
  - the bit is flipped if the next lower bit is changed from 1 to 0



Doubly-Exponential Grid in $\mathcal{SHOIQ}^{\sqcap}$

- $\mathcal{SHOIQ}$ does not have a tree model property
  - It allows to bound the cardinality of concepts using nominals—one element sets: $A \sqsubseteq o_1 \sqcup \ldots \sqcup o_n$.

DOUBLY-EXPONENTIAL GRID IN $\mathcal{SHOIQ}^{\sqcap}$

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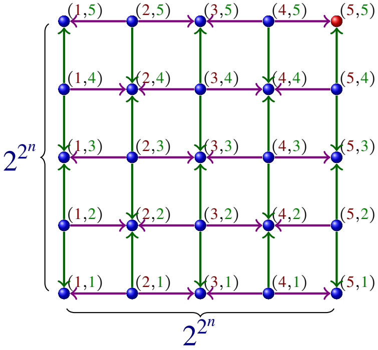
  - It allows to bound the cardinality of concepts using nominals—one element sets: $A \sqsubseteq o_1 \sqcup \ldots \sqcup o_n$.
- Using nominals it is possible to express a grid in $\mathcal{SHOIQ}$:
  - use two counters to encode the coordinates of the grid
  - increment / copy the counters over respective roles



Doubly-Exponential Grid in $\mathcal{SHOIQ}^{\sqcap}$

- $\mathcal{SHOIQ}$ does not have a tree model property
  - It allows to bound the cardinality of concepts using nominals—one element sets: $A \sqsubseteq o_1 \sqcup \ldots \sqcup o_n$.
- Using nominals it is possible to express a grid in $\mathcal{SHOIQ}$:
  - use two counters to encode the coordinates of the grid
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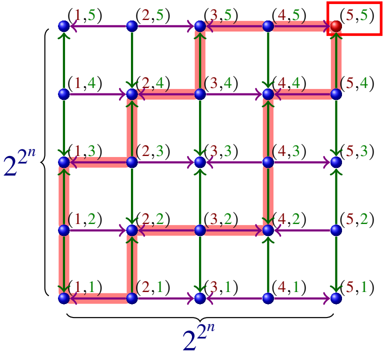

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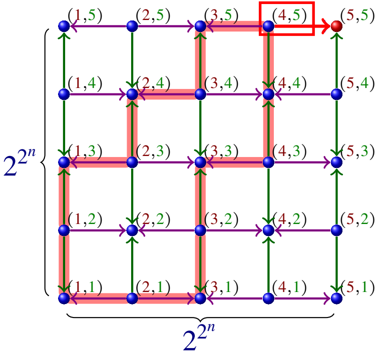

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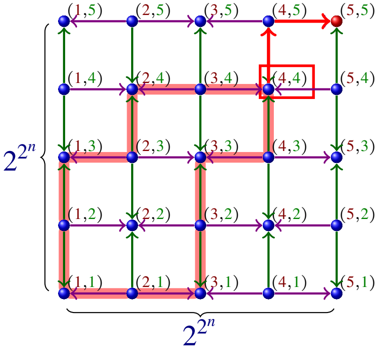

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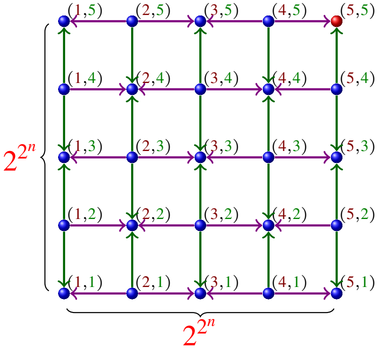

- For $\mathcal{SHOIQ}^{\sqcap}$ use doubly-exponential counters

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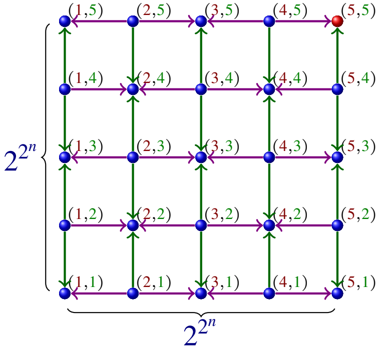
THEOREM

(Finite model) reasoning in $\mathcal{SHOIF}^{\sqcap}$ (and therefore in $\mathcal{SHOIQ}^{\sqcap}$) is N2ExpTime-hard.



Outline

Summary

- New complexity results:
  - $\mathcal{SHQ}^{\sqcap}$ is ExpTime-complete;
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- Thank you for your attention!