Unchain My $\mathcal{EL}$ Reasoner!

Optimized reasoning with role chains in $\mathcal{ELR}$

Yevgeny Kazakov   Markus Krötzsch   František Simančík

Oxford University Computing Laboratory

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Consequence-Based Procedure for $\mathcal{ELR}$

Normalized axioms:

$$B_1 \sqcap \cdots \sqcap B_n \sqsubseteq D$$

$$B \sqsubseteq \exists R.C$$

$$\exists R.C \sqsubseteq D$$

$$R \sqsubseteq S$$

$$R \cdot S \sqsubseteq H$$

Consequence-Based Procedure for $\mathcal{ELR}$

I1 $\dfrac{}{A \sqsubseteq A}$

CR1 $\dfrac{A \sqsubseteq B_1 \quad \cdots \quad A \sqsubseteq B_n}{A \sqsubseteq D} : \quad B_1 \sqcap \cdots \sqcap B_n \sqsubseteq D \in \mathcal{O}$

CR2 $\dfrac{A \sqsubseteq B}{A \sqsubseteq \exists R.C} : \quad B \sqsubseteq \exists R.C \in \mathcal{O}$

CR3 $\dfrac{A \sqsubseteq \exists R.B \quad B \sqsubseteq C}{A \sqsubseteq D} : \quad \exists R.C \sqsubseteq D \in \mathcal{O}$

CR4 $\dfrac{A \sqsubseteq \exists R.B}{A \sqsubseteq \exists S.B} : \quad R \sqsubseteq S \in \mathcal{O}$

CR5 $\dfrac{A \sqsubseteq \exists R.B \quad B \sqsubseteq \exists S.C}{A \sqsubseteq \exists H.C} : \quad R \cdot S \sqsubseteq H \in \mathcal{O}$

The Source of Inefficiency

$A_1 \sqsubseteq \exists \mathsf{isPartOf}.A_2$

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- Left-linear closure computation:
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- Questions:
  - How to check if a role chain axiom can be used in a left-linear way?
  - How to identify such role chain axioms?

OUR RESULTS

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1. Formalized the notion of left-admissibility for a set of role chain axioms — “can be safely used in a left-linear way”.
2. Proved a necessary and sufficient condition for left-admissibility that can be checked in polynomial time.
3. Implemented the optimized procedure and measured the number of rule applications for GO, FMA, and GALEN.

Left-admissible role chain axioms

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Definition

A set or role chain axioms is left-admissible if for every role chain $R_1 \cdot R_2 \cdots R_n$ and every role S we have:

$$R_1 \cdot R_2 \cdots R_n \sqsubseteq^* S \quad \text{implies} \quad ((R_1 \cdot R_2) \cdots R_n) \sqsubseteq^* S$$

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- The condition for left-admissibility is not effective:

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- It is sufficient to check the condition only for triplets:

THEOREM

If for every triplet chain $R_1 \cdots R_3$ and every role S, we have:

$$R_1 \cdot (R_2 \cdot R_3) \sqsubseteq^* S \quad \textit{implies} \quad (R_1 \cdot R_2) \cdot R_3 \sqsubseteq^* S,$$

then the set of role chain axioms is left-admissible.

LEFT-ADMISSIBLE SUBSETS OF CHAIN AXIOMS

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Left-admissible subsets of chain axioms

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- How to compute such $\mathcal{L}$ (preferably large)?
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  - Start with $\mathcal{L} = \emptyset$, expand using (a), then (b).

Experiments

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Size of the input (normalized):					
$R \cdot S \sqsubseteq H$	1	1	58	385	385
other	30,696	134,067	106,607	129,281	912,020

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Number of inferences after:					
CR5	19,756	1,186,733	216,982	1,087,328	8,728,711
reduction	43% (5%)	78% (65%)	21% (1%)	14% (2%)	26% (4%)

Conclusions

- Left-linear optimization of rule using role chain axioms
- Reduces the number of inferences while preserving the consequences
- Left-admissibility can be effectively computed
- Relatively easy to implement
- Does not improve running times as much as expected