Semantic Technology Tutorial

Part 2: Logical Foundations
What Is OWL?

A Description Logic (DL) with a web-friendly syntax
What Are Description Logics?
What Are Description Logics?

Decidable fragments of First Order Logic

Thank you for listening

Any questions?
Crash Course in FOL
Crash Course in (simplified) FOL

• Syntax
  – Non-logical symbols (signature)
    • Constants: Felix, MyMat
    • Predicates(arity): Animal(1), Cat(1), has-color(2), sits-on(2)
  – Logical symbols:
    • Variables: x, y
    • Operators: ∧, ∨, →, ¬, …
    • Quantifiers: ∃, ∀
    • Equality: =
  – Formulas:
    • Cat(Felix), Mat(MyMat), sits-on(Felix, MyMat)
    • Cat(x), Cat(x) ∨ Human(x), ∃y. Mat(y) ∧ sits-on(x, y)
    • ∀x. Cat(x) → Animal(x), ∀x. Cat(x) → (∃y. Mat(y) ∧ sits-on(x, y))

Formula with no free variables often called a sentence
Crash Course in (simplified) FOL

- Semantics
Crash Course in (simplified) FOL

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• Semantics

Why should I care about semantics? -- In fact I heard that a little goes a long way!
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Well, from a philosophical POV, we need to specify the relationship between statements in the logic and the existential phenomena they describe.
Crash Course in (simplified) FOL

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Why should I care about semantics? -- In fact I heard that a little goes a long way!

Well, from a philosophical POV, we need to specify the relationship between statements in the logic and the existential phenomena they describe.

That’s OK, but I don’t get paid for philosophy.
Crash Course in (simplified) FOL

• Semantics

Why should I care about semantics? -- In fact I heard that a little goes a long way!

Well, from a philosophical POV, we need to specify the relationship between statements in the logic and the existential phenomena they describe.

That’s OK, but I don’t get paid for philosophy.

From a practical POV, in order to specify, build and test (ontology-based) tools/systems we need to precisely define relationships (like entailment) between logical statements – this defines the intended behaviour of tools/systems.
Crash Course in (simplified) FOL

• Semantics

In FOL we define the semantics in terms of models (a model theory). A model is supposed to be an analogue of (part of) the world being modeled. FOL uses a very simple kind of model, in which “objects” in the world (not necessarily physical objects) are modeled as elements of a set, and relationships between objects are modeled as sets of tuples.
In FOL we define the semantics in terms of models (a model theory). A model is supposed to be an analogue of (part of) the world being modeled. FOL uses a very simple kind of model, in which “objects” in the world (not necessarily physical objects) are modeled as elements of a set, and relationships between objects are modeled as sets of tuples.

Note that this is exactly the same kind of model as used in a database: objects in the world are modeled as values (elements) and relationships as tables (sets of tuples).
Crash Course in (simplified) FOL

• Semantics
  – Model: a pair \( \langle D, , I \rangle \) with \( D \) a non-empty set and \( I \) an interpretation
    • \( C^I \) is an element of \( D \) for \( C \) a constant
    • \( v^I \) is an element of \( D \) for \( v \) a variable
    • \( P^I \) is a subset of \( D^n \) for \( P \) a predicate of arity \( n \)
  – E.g., \( D = \{ a, b, c, d, e, f \} \), and
    \[
    \begin{align*}
    \text{Felix}^I &= a \\
    \text{MyMat}^I &= b \\
    \text{Cat}^I &= \{ a, c \} \\
    \text{Mat}^I &= \{ b, e \} \\
    \text{Animal}^I &= \{ a, c, d \} \\
    \text{sits-on}^I &= \{ \langle a, b \rangle, \langle c, e \rangle \}
    \end{align*}
    \]
Crash Course in (simplified) FOL

• Semantics
  – Evaluation: truth value in a given model $M = \langle D, \cdot^I \rangle$
    • $P(t_1, \ldots, t_n)$ is true iff $\langle t_1^I, \ldots, t_n^I \rangle \in P^I$
    • $A \land B$ is true iff $A$ is true and $B$ is true
    • $\neg A$ is true iff $A$ is not true
  – E.g.,
    \begin{align*}
    \text{Cat(Felix)} & \quad \text{true} \\
    \text{Cat(MyMat)} & \quad \text{false} \\
    \neg \text{Mat(Felix)} & \quad \text{true} \\
    \text{sits-on(Felix, MyMat)} & \quad \text{true} \\
    \text{Mat(Felix)} \lor \text{Cat(Felix)} & \quad \text{true} \\
    \text{Cat(Felix)} \lor \text{Animal(Felix)} & \quad \text{true}
    \end{align*}

\[
D = \{a, b, c, d, e, f\} \\
Felix^I = a \\
MyMat^I = b \\
\text{Cat}^I = \{a, c\} \\
\text{Mat}^I = \{b, e\} \\
\text{Animal}^I = \{a, c, d\} \\
\text{sits-on}^I = \{\langle a, b \rangle, \langle c, e \rangle\}
Crash Course in (simplified) FOL

- **Semantics**
  - Evaluation: truth value in a given model $M = \langle D, \cdot^I \rangle$
    - $\exists x. A$ is *true* iff exists $\cdot^{I'}$ s.t. $\cdot^I$ and $\cdot^{I'}$ differ only w.r.t. $x$, and $A$ is *true* w.r.t. $\langle D, \cdot^{I'} \rangle$
    - $\forall x. A$ is *true* iff for all $\cdot^{I'}$ s.t. $\cdot^I$ and $\cdot^{I'}$ differ only w.r.t. $x$, $A$ is *true* w.r.t. $\langle D, \cdot^{I'} \rangle$

E.g.,

$\exists x. \text{Cat}(x)$

$\forall x. \text{Cat}(x)$

$\exists x. \text{Cat}(x) \land \text{Mat}(x)$

$\forall x. \text{Cat}(x) \rightarrow \text{Animal}(x)$

$\forall x. \text{Cat}(x) \rightarrow (\exists y. \text{Mat}(y) \land \text{sits-on}(x, y))$

\[ D = \{a, b, c, d, e, f\} \]

\[ \text{Felix}^I = a \]

\[ \text{MyMat}^I = b \]

\[ \text{Cat}^I = \{a, c\} \]

\[ \text{Mat}^I = \{b, e\} \]

\[ \text{Animal}^I = \{a, c, d\} \]

\[ \text{sits-on}^I = \{\langle a, b\rangle, \langle c, e\rangle\} \]
Crash Course in (simplified) FOL

- Semantics
  - Given a model $M$ and a formula $F$, $M$ is a model of $F$ (written $M \models F$) iff $F$ evaluates to true in $M$.
  - A formula $F$ is **satisfiable** iff there exists a model $M$ s.t. $M \models F$.
  - A formula $F$ **entails** another formula $G$ (written $F \models G$) iff every model of $F$ is also a model of $G$ (i.e., $M \models F$ implies $M \models G$).

E.g.,

$$M \models \exists x. \text{Cat}(x)$$
$$M \not\models \forall x. \text{Cat}(x)$$
$$M \not\models \exists x. \text{Cat}(x) \land \text{Mat}(x)$$
$$M \models \forall x. \text{Cat}(x) \rightarrow \text{Animal}(x)$$
$$M \models \forall x. \text{Cat}(x) \rightarrow (\exists y. \text{Mat}(y) \land \text{sits-on}(x, y))$$

$$D = \{a, b, c, d, e, f\}$$
$$\text{Felix}^I = a$$
$$\text{MyMat}^I = b$$
$$\text{Cat}^I = \{a, c\}$$
$$\text{Mat}^I = \{b, e\}$$
$$\text{Animal}^I = \{a, c, d\}$$
$$\text{sits-on}^I = \{\langle a, b\rangle, \langle c, e\rangle\}$$
Crash Course in (simplified) FOL

• Semantics
  – Given a model $M$ and a formula $F$, $M$ is a model of $F$ (written $M \vDash F$) iff $F$ evaluates to true in $M$
  – A formula $F$ is **satisfiable** iff there exists a model $M$ s.t. $M \vDash F$
  – A formula $F$ **entails** another formula $G$ (written $F \vDash G$) iff **every** model of $F$ is also a model of $G$ (i.e., $M \vDash F$ implies $M \vDash G$)

E.g.,

✓ $\text{Cat}(\text{Felix}) \vDash \exists x. \text{Cat}(x)$  (Cat(\text{Felix}) \wedge \neg \exists x. \text{Cat}(x) \text{ is not satisfiable})
✓ $(\forall x. \text{Cat}(x) \rightarrow \text{Animal}(x)) \wedge \text{Cat}(\text{Felix}) \vDash \text{Animal}(\text{Felix})$
✓ $(\forall x. \text{Cat}(x) \rightarrow \text{Animal}(x)) \wedge \neg \text{Animal}(\text{Felix}) \vDash \neg \text{Cat}(\text{Felix})$
✗ $\text{Cat}(\text{Felix}) \vDash \forall x. \text{Cat}(x)$
✗ $\text{sits-on}(\text{Felix, Mat1}) \wedge \text{sits-on}(\text{Tiddles, Mat2}) \vDash \neg \text{sits-on}(\text{Felix, Mat2})$
✗ $\text{sits-on}(\text{Felix, Mat1}) \wedge \text{sits-on}(\text{Tiddles, Mat1}) \vDash \exists x^2 \text{sits-on}(x, \text{Mat1})$
✗ $\vDash \forall x. \text{Cat}(x) \rightarrow \text{Animal}(x)$  a **tautology**
Decidable Fragments

• FOL (satisfiability) well known to be undecidable
  – A sound, complete and terminating algorithm is impossible

• Interesting decidable fragments include, e.g.,
  – C2: FOL with 2 variables and Counting quantifiers ($\exists^{\geq n}, \exists^{\leq n}$)
    • Counting quantifiers abbreviate pairwise (in-) equalities, e.g.:
      $\exists^{\geq 3} x. \text{Cat}(x)$ equivalent to
      $\exists x, y, z. \text{Cat}(x) \land \text{Cat}(y) \land \text{Cat}(z) \land x \neq y \land x \neq z \land y \neq z$
      $\exists^{\leq 2} x. \text{Cat}(x)$ equivalent to
      $\forall x, y, z. \text{Cat}(x) \land \text{Cat}(y) \land \text{Cat}(z) \rightarrow x = y \lor x = z \lor y = z$
  – Propositional modal and description logics
  – Guarded fragment
Description Logics
What Are Description Logics?

- A family of logic based Knowledge Representation formalisms
  - Originally descended from semantic networks and KL-ONE
  - Describe domain in terms of concepts (aka classes), roles (aka properties, relationships) and individuals

```
Animal

Cat  has-color  Black

Felix  sits-on  Mat
```

[Quillian, 1967]
What Are Description Logics?

• Modern DLs (after Baader et al) distinguished by:
  – Fully fledged logics with formal semantics
    • Decidable fragments of FOL (often contained in $C_2$)
    • Closely related to Propositional Modal/Dynamic Logics & Guarded Fragment
  – Computational properties well understood (worst case complexity)
  – Provision of inference services
    • Practical decision procedures (algorithms) for key problems
      (satisfiability, subsumption, query answering, etc)
    • Implemented systems (highly optimised)
  • The basis for widely used ontology languages
Web Ontology Language OWL (2)

- **W3C recommendation(s)**
- Motivated by **Semantic Web** activity
  - Add meaning to web content by annotating it with terms defined in ontologies
- Supported by **tools and infrastructure**
  - APIs (e.g., OWL API, Thea, OWLink)
  - Development environments (e.g., Protégé, Swoop, TopBraid Composer, Neon)
  - Reasoners & Information Systems (e.g., HermiT, RDFox, FaCT++, Pellet, ELK, Ontop, …)
- Based on **Description Logics** *(SHOIN / SROIQ)*
DL Syntax

• Signature
  – **Concept** (aka class) names, e.g., Cat, Animal, Doctor
    • Equivalent to FOL unary predicates
  – **Role** (aka property) names, e.g., sits-on, hasParent, loves
    • Equivalent to FOL binary predicates
  – **Individual** names, e.g., Felix, John, Mary, Boston, Italy
    • Equivalent to FOL constants
DL Syntax

• Operators
  – Many kinds available, e.g.,
    • Standard FOL Boolean operators (\(\cap, \cup, \neg\))
    • Restricted form of quantifiers (\(\exists, \forall\))
    • Counting (\(\geq, \leq, =\))
    • ...
DL Syntax

• Concept expressions, e.g.,
  – Doctor ⊔ Lawyer
  – Rich ⊓ Happy
  – Cat ⊓ ∃sits-on.Mat

• Equivalent to FOL formulae with one free variable
  – Doctor(\(x\)) ∨ Lawyer(\(x\))
  – Rich(\(x\)) ∧ Happy(\(x\))
  – ∃y.(Cat(\(x\)) ∧ sits-on(\(x, y\)))
DL Syntax

• Special concepts
  – $\top$ (aka top, Thing, most general concept)
  – $\bot$ (aka bottom, Nothing, inconsistent concept)

used as abbreviations for
  – $(A \cup \neg A)$ for any concept $A$
  – $(A \cap \neg A)$ for any concept $A$
DL Syntax

• Role expressions, e.g.,
  - loves
  - hasParent ◦ hasBrother

• Equivalent to FOL formulae with two free variables
  - loves(y, x)
  - ∃z.(hasParent(x, z) ∧ hasBrother(z, y))
DL Syntax

• “Schema” Axioms, e.g.,
  - Rich $\subseteq \neg$Poor (concept inclusion)
  - Cat $\sqcap \exists$sits-on.Mat $\subseteq$ Happy (concept inclusion)
  - BlackCat $\equiv$ Cat $\sqcap \exists$hasColour.BLACK (concept equivalence)
  - sits-on $\subseteq$ touches (role inclusion)
  - Trans(part-of) (transitivity)

• Equivalent to (particular form of) FOL sentence, e.g.,
  - $\forall x.(\text{Rich}(x) \rightarrow \neg\text{Poor}(x))$
  - $\forall x.(\text{Cat}(x) \land \exists y.(\text{sits-on}(x,y) \land \text{Mat}(y)) \rightarrow \text{Happy}(x))$
  - $\forall x.(\text{BlackCat}(x) \leftrightarrow (\text{Cat}(x) \land \exists y.(\text{hasColour}(x,y) \land \text{Black}(y))))$
  - $\forall x,y.(\text{sits-on}(x,y) \rightarrow \text{touches}(x,y))$
  - $\forall x,y,z.((\text{sits-on}(x,y) \land \text{sits-on}(y,z)) \rightarrow \text{sits-on}(x,z))$
DL Syntax

• “Data” **Axioms** (aka Assertions or Facts), e.g.,
  – BlackCat(Felix) (concept assertion)
  – Mat(Mat1) (concept assertion)
  – Sits-on(Felix,Mat1) (role assertion)

• Directly equivalent to FOL “ground facts”
  – Formulae with no variables
DL Syntax

• A set of axioms is called a **TBox**, e.g.:

\[
\{\text{Doctor} \sqsubseteq \text{Person}, \\
\text{Parent} \equiv \text{Person} \sqcap \exists \text{hasChild}.\text{Person}, \\
\text{HappyParent} \equiv \text{Parent} \sqcap \forall \text{hasChild}.\text{Person}\}
\]

Note
Facts sometimes written

\[
\{\text{HappyParent(John)}, \\
\text{hasChild(John,Mary)}\}
\]

• A set of facts is called an **ABox**

• A **Knowledge Base** (KB) is just a TBox plus an Abox
  – Often written \( \mathcal{K} = \langle T, A \rangle \)
The DL Family

• Many different DLs, often with “strange” names
  – E.g., $EL$, $ALC$, $SHIQ$

• Particular DL defined by:
  – Concept operators ($\cap$, $\cup$, $\neg$, $\exists$, $\forall$, etc.)
  – Role operators ($\cdot$, $\circ$, etc.)
  – Concept axioms ($\sqsubseteq$, $\equiv$, etc.)
  – Role axioms ($\sqsubseteq$, Trans, etc.)
The DL Family

• E.g., \( \mathcal{EL} \) is a well known “sub-Boolean” DL
  – Concept operators: \( \sqcap, \neg, \exists \)
  – No role operators (only atomic roles)
  – Concept axioms: \( \sqsubseteq, \equiv \)
  – No role axioms

• E.g.:

  \[ \text{Parent} \equiv \text{Person} \sqcap \exists \text{hasChild}.\text{Person} \]
The DL Family

• *ALC* is the smallest propositionally closed DL
  – Concept operators: \( \cap, \cup, \neg, \exists, \forall \)
  – No role operators (only atomic roles)
  – Concept axioms: \( \sqsubseteq, \equiv \)
  – No role axioms

• E.g.:

\[
ProudParent \equiv Person \sqsubseteq \forall \text{hasChild.}(Doctor \sqcup \exists \text{hasChild.Doctor})
\]
The DL Family

- **S** used for **ALC** extended with (role) transitivity axioms
- **Additional letters** indicate various extensions, e.g.:
  - **H** for role hierarchy (e.g., hasDaughter $\sqsubseteq$ hasChild)
  - **R** for role box (e.g., hasParent $\pm$ hasBrother $\sqsubseteq$ hasUncle)
  - **O** for nominals/singleton classes (e.g., \{Italy\})
  - **I** for inverse roles (e.g., isChildOf $\equiv$ hasChild$^-$)
  - **N** for number restrictions (e.g., $\geq 2$ hasChild, $\leq 3$ hasChild)
  - **Q** for qualified number restrictions (e.g., $\geq 2$ hasChild.Doctor)
  - **F** for functional number restrictions (e.g., $\leq 1$ hasMother)
- E.g., **SHIQ** = **S** + role hierarchy + inverse roles + QNRs
## DL Naming Schemes

<table>
<thead>
<tr>
<th>Name</th>
<th>Syntax</th>
<th>Sym</th>
<th>$\mathcal{AL}$</th>
<th>$\mathcal{EL}$</th>
<th>$\mathcal{S}$</th>
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<td>✔</td>
<td>✔</td>
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<tr>
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<td>$\bot$</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
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<td>Conjunction</td>
<td>$C \sqcap D$</td>
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<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Atomic negation</td>
<td>$\neg A$</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
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<td>Value restr.</td>
<td>$\forall r. C$</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Disjunction</td>
<td>$C \sqcup D$</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
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<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
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<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
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<td>Unqualified number restr.</td>
<td>$(\leq n \cdot r)$</td>
<td>$\mathcal{N}$</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Qualified number restr.</td>
<td>$(\geq n \cdot r)$</td>
<td>$\mathcal{Q}$</td>
<td>✔</td>
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</tr>
<tr>
<td>Nominal</td>
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<td>$\mathcal{O}$</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Inverse role</td>
<td>$r^{-}$</td>
<td>$\mathcal{I}$</td>
<td>✔</td>
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<td>✔</td>
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<tr>
<td>Role inclusion</td>
<td>$r \sqsubseteq s$</td>
<td>$\mathcal{H}$</td>
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<td>✔</td>
<td>✔</td>
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<tr>
<td>Complex role inclusion</td>
<td>$r_1 \circ \ldots \circ r_n \sqsubseteq s$</td>
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</table>
The DL Family

• Numerous other extensions have been investigated
  – Concrete domains (numbers, strings, etc)
  – DL-safe rules (Datalog-like rules)
  – Fixpoints
  – Role value maps
  – Additional role constructors (\(\cap, \cup, \neg, \circ, \text{id}, \ldots\))
  – Nary (i.e., predicates with arity >2)
  – Temporal
  – Fuzzy
  – Probabilistic
  – Non-monotonic
  – Higher-order
  – …
DL Semantics

Via translation to FOL, or directly using FO model theory:

Interpretation function $\mathcal{I}$

- **Individuals** $i^\mathcal{I} \in \Delta^\mathcal{I}$
  - John
  - Mary

- **Concepts** $C^\mathcal{I} \subseteq \Delta^\mathcal{I}$
  - Lawyer
  - Doctor
  - Vehicle

- **Roles** $r^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$
  - hasChild
  - owns

Interpretation domain $\Delta^\mathcal{I}$
DL Semantics: Concepts

- Interpretation function extends to **concept expressions** in the obvious(ish) way, e.g.:

\[
\begin{align*}
(C \cap D)^I &= C^I \cap D^I \\
(C \cup D)^I &= C^I \cup D^I \\
(\neg C)^I &= \Delta^I \setminus C^I \\
\{x\}^I &= \{x^I\} \\
(\exists R.C)^I &= \{x \mid \exists y. \langle x, y \rangle \in R^I \land y \in C^I\} \\
(\forall R.C)^I &= \{x \mid \forall y. \langle x, y \rangle \in R^I \Rightarrow y \in C^I\} \\
(\leq nR)^I &= \{x \mid \#\{y \mid \langle x, y \rangle \in R^I\} \leq n\} \\
(\geq nR)^I &= \{x \mid \#\{y \mid \langle x, y \rangle \in R^I\} \geq n\}
\end{align*}
\]
## DL Semantics: Concepts

<table>
<thead>
<tr>
<th>Name</th>
<th>Syntax</th>
<th>Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>$T$</td>
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<td>Conjunction</td>
<td>$C \sqcap D$</td>
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<tr>
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<td>$C^T \sqcup D^T$</td>
</tr>
<tr>
<td>Negation</td>
<td>$\neg C$</td>
<td>$\Delta^T \setminus C^T$</td>
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<tr>
<td>Exist. restr.</td>
<td>$\exists r.C$</td>
<td>${d \in \Delta^T \mid \exists e \in \Delta^T, (d,e) \in r^T \land e \in C^T}$</td>
</tr>
<tr>
<td>Value restr.</td>
<td>$\forall r.C$</td>
<td>${d \in \Delta^T \mid \forall e \in \Delta^T, (d,e) \in r^T \rightarrow e \in C^T}$</td>
</tr>
<tr>
<td>Self restr.</td>
<td>$\exists r.\text{Self}$</td>
<td>${d \in \Delta^T \mid (d,d) \in r^T}$</td>
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<tr>
<td>Unqualified number restr.</td>
<td>$\langle \leq n \rangle$</td>
<td>${d \in \Delta^T \mid #{e \mid (d,e) \in r^T} \leq n}$</td>
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<td>Qualified number restr.</td>
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<td>${a^T}$</td>
</tr>
<tr>
<td>Role value map</td>
<td>$(r \sqsubseteq s)$</td>
<td>${d \in \Delta^T \mid {e \mid (d,e) \in r^T} = {e' \mid (d,e') \in s^T}}$</td>
</tr>
<tr>
<td>Predicate restr.</td>
<td>$\exists c_1, \ldots, c_k. P$</td>
<td>${d \in \Delta^T \mid (c_1^T(d), \ldots, c_k^T(d)) \in P^D}$</td>
</tr>
<tr>
<td>Role composition</td>
<td>$r \circ s$</td>
<td>${(d, f) \in \Delta^T \times \Delta^T \mid \exists e \in \Delta^T, (d,e) \in r^T \land (e,f) \in s^T}$</td>
</tr>
<tr>
<td>Inverse role</td>
<td>$r^{-}$</td>
<td>${(e, d) \in \Delta^T \times \Delta^T \mid (d,e) \in r^T}$</td>
</tr>
<tr>
<td>Feature chain</td>
<td>$g_1 \cdots g_nh$</td>
<td>$(g_1 \cdots g_nh)^T(d) = h^T(g_n^T(\cdots (g_1^T(d)) \cdots))$</td>
</tr>
</tbody>
</table>
DL Semantics: Axioms

- Given a model $M = \langle D, \cdot^I \rangle$
  - $M \models C \subseteq D$ iff $C^I \subseteq D^I$
  - $M \models C \equiv D$ iff $C^I = D^I$
  - $M \models C(a)$ iff $a^I \in C^I$
  - $M \models R(a, b)$ iff $\langle a^I, b^I \rangle \in R^I$
  - $M \models \langle T, A \rangle$ iff for every axiom $ax \in T \cup A$, $M \models ax$
## DL Semantics: Axioms

<table>
<thead>
<tr>
<th>Name</th>
<th>Syntax</th>
<th>Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>General concept inclusion</td>
<td>$C \subseteq D$</td>
<td>$C^\mathcal{I} \subseteq D^\mathcal{I}$</td>
</tr>
<tr>
<td>Concept definition</td>
<td>$A \equiv C$</td>
<td>$A^\mathcal{I} = C^\mathcal{I}$</td>
</tr>
<tr>
<td>Role inclusion</td>
<td>$r \subseteq s$</td>
<td>$r^\mathcal{I} \subseteq s^\mathcal{I}$</td>
</tr>
<tr>
<td>Role disjointness</td>
<td>$\text{Disj}(r, s)$</td>
<td>$r^\mathcal{I} \cap s^\mathcal{I} = \emptyset$</td>
</tr>
<tr>
<td>Role transitivity</td>
<td>$\text{Trans}(r)$</td>
<td>$r^\mathcal{I}$ is transitive</td>
</tr>
<tr>
<td>Role functionality</td>
<td>$\text{Func}(r)$</td>
<td>$r^\mathcal{I}$ is functional</td>
</tr>
<tr>
<td>Role reflexivity</td>
<td>$\text{Ref}(r)$</td>
<td>$r^\mathcal{I}$ is reflexive</td>
</tr>
<tr>
<td>Role irreflexivity</td>
<td>$\text{Irref}(r)$</td>
<td>$r^\mathcal{I}$ is irreflexive</td>
</tr>
<tr>
<td>Role symmetry</td>
<td>$\text{Sym}(r)$</td>
<td>$r^\mathcal{I}$ is symmetrical</td>
</tr>
<tr>
<td>Role antisymmetry</td>
<td>$\text{Asym}(r)$</td>
<td>$r^\mathcal{I}$ is antisymmetrical</td>
</tr>
<tr>
<td>Concept assertion</td>
<td>$a : C$</td>
<td>$a^\mathcal{I} \in C^\mathcal{I}$</td>
</tr>
<tr>
<td>Role assertion</td>
<td>$(a, b) : r$</td>
<td>$(a^\mathcal{I}, b^\mathcal{I}) \in r^\mathcal{I}$</td>
</tr>
</tbody>
</table>
DL Semantics: Conjunctive Queries

Given sets $V$ of variables and $I$ of individuals, a **conjunctive query** (CQ) $q$ has the form

$$\exists v_1 \ldots \exists v_n (\alpha_1 \land \ldots \land \alpha_n)$$

where $v_i \in V$ and each $\alpha_i$ is either a concept atom $C(t)$ or a role atom $r(t, t')$, and where $t, t'$ are terms, i.e., elements of $V \cup I$.

The variables $v_1 \ldots v_n$ are called **quantified variables**, and all other variables in $q$ are called **answer variables**; we often write $q(\vec{x})$ to denote that $\vec{x}$ are the answer variables in $q$, and we often omit the existential quantifiers, e.g.:

$$q(x, z) = C(x) \land r(x, y) \land r(y, z) \land D(z)$$

The arity of a CQ $q$ is the number of answer variables in $q$; a CQ of arity zero is called a **Boolean CQ**.
DL Semantics: Conjunctive Queries

Given model $M = \langle D, \cdot^I \rangle$, and a CQ $q$ with answer variables $v_1 \ldots v_k$

- $(a_1, \ldots, a_k)$ is an answer to $q$ in $M$ if $\{a_1, \ldots, a_k\} \subseteq I$, and we can extend $\cdot^I$ to the variables in $q$ such that:
  - $v_i^I = a_i^I$ for $i \leq 1 \leq k$;
  - for each concept atom $C(t)$ in $q$, we have $t^I \in C^I$; and
  - for each role atom $r(t_1, t_2)$ in $q$ we have $(t_1^I, t_2^I) \in r^I$.
- We use $\text{ans}(q, M)$ to denote the set of all answers to $q$ in $M$. 
DL Semantics: Conjunctive Queries

Given a KB $\mathcal{K}$, and a CQ $q$ with answer variables $v_1 \ldots v_k$

- $(a_1, \ldots, a_k)$ is a certain answer to $q$ in $\mathcal{K}$ if:
  - $a_1 \ldots a_k$ are individuals occurring in $\mathcal{K}$; and
  - $(a_1, \ldots, a_k) \in \text{ans}(q, M)$ for every model $M$ of $\mathcal{K}$.

- We use $\text{cert}(q, \mathcal{K})$ to denote the set of all certain answers to $q$ in $\mathcal{K}$.

- If $q$ is a Boolean CQ, then we say that $\mathcal{K}$ entails $q$ (written $\mathcal{K} \models q$) if the empty tuple is a certain answer to $q$ in $\mathcal{K}$.
DL Semantics: Reasoning Problems

Given a knowledge base $\mathcal{K}$, and concepts $C, D$:

- **KB consistency**: $\mathcal{K}$ is consistent if there exists some model $M$ s.t. $M \models \mathcal{K}$

- **Concept satisfiability**: $C$ is satisfiable w.r.t. $\mathcal{K}$ if there exists a model $M = \langle D, \cdot^\mathcal{I} \rangle$ of $\mathcal{K}$ with $C^\mathcal{I} \neq \emptyset$

- **Concept subsumption**: $C$ is subsumed by $D$ w.r.t. $\mathcal{K}$, written $\mathcal{K} \models C \sqsubseteq D$, if $C^\mathcal{I} \subseteq D^\mathcal{I}$ in every model $\mathcal{I}$ of $\mathcal{K}$

- **Axiom entailment**: An axiom $A$ is entailed by $\mathcal{K}$ (written $\mathcal{K} \models A$) if for every model $M$ of $\mathcal{K}$, $M \models A$

- **CQ answering**: Given a KB $\mathcal{K}$ and a CQ $q$, compute $\text{cert}(q, \mathcal{K})$
DL Semantics: Reasoning Problems

Note that many problems are inter-reducible. For a KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$, concepts $C, D$, a role $r$ and an individual $a$ that does not occur in $\mathcal{K}$:

- $\mathcal{K}$ is consistent iff $C \sqcup \neg C$ is satisfiable w.r.t. $\mathcal{K}$
- $C$ is satisfiable w.r.t. $\mathcal{K}$ iff $\langle \mathcal{T}, \mathcal{A} \cup \{ a : C \} \rangle$ is consistent
- $\mathcal{K} \models C \sqsubseteq D$ iff $\langle \mathcal{T}, \mathcal{A} \cup \{ a : (C \sqcap \neg D) \} \rangle$ is not consistent
- $\mathcal{K} \models a : C$ iff $\langle \mathcal{T}, \mathcal{A} \cup \{ a : \neg C \} \rangle$ is not consistent
- $a$ is an answer to $q(x) = C(x) \land r(x, y) \land D(y)$ in $\mathcal{K}$ iff $\mathcal{K} \models a : (C \sqcap \exists r.D)$

CQs are not in general reducible to “standard” reasoning problems, but tree shaped CQs can be so reduced via rolling up as in the last example above.
DL Semantics: Examples

E.g.,

\[ T = \{ \text{Doctor} \sqsubseteq \text{Person}, \text{Parent} \equiv \text{Person} \sqcap \exists \text{hasChild}.\text{Person}, \]
\[ \text{HappyParent} \equiv \text{Parent} \sqcap \forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor}) \} \]
\[ A = \{ \text{John:HappyParent}, \text{John hasChild Mary}, \text{John hasChild Sally}, \]
\[ \text{Mary:}\neg\text{Doctor}, \text{Mary hasChild Peter}, \text{Mary}:(\leq 1 \text{ hasChild}) \]
DL and FOL

• Most DLs are subsets of C2
  – But reduction to C2 may be (highly) non-trivial
    • Trans(R) naively reduces to $\forall x, y, z. R(x, y) \land R(y, z) \rightarrow R(x, z)$

• Why use DL instead of C2?
  – Syntax is succinct and convenient for KR applications
  – Syntactic conformance guarantees being inside C2
    • Even if reduction to C2 is non-obvious
  – Different combinations of constructors can be selected
    • To guarantee decidability
    • To reduce complexity
  – DL research has mapped out the decidability/complexity landscape in great detail
    • See Evgeny Zolin’s DL Complexity Analyzer
      http://www.cs.man.ac.uk/~ezolin/dl/
Complexity of reasoning in Description Logics

Note: the information here is (always) incomplete and updated often

Base description logic: Atributive Language with Complements

\[ \text{ALC} ::= \bot \mid A \mid \neg C \mid C \land D \mid C \lor D \mid \exists R.C \mid \forall R.C \]

**Concept constructors:**
- \( T \): functionality (\( \leq 1 \) \( R \))
- \( \mathcal{N} \): (unqualified) number restrictions (\( \geq n \) \( R \)), (\( \leq n \) \( R \))
- \( Q \): qualified number restrictions (\( \geq n \) \( R.C \)), (\( \leq n \) \( R.C \))
- \( O \): nominals: \( \{a\} \) or \( \{a_1, \ldots, a_n\} \) ("one-of" constructor)
- \( \mu \): least fixpoint operator: \( \mu X.C \)
- \( RCS \): role-value-maps
- \( f = g \): agreement of functional role chains ("same-as")

**Role constructors:**
- \( I \): role inverses: \( R^- \)
- \( \cap \): role intersection: \( R \cap S \)
- \( U \): role union: \( R \cup S \)
- \( \neg \): role complement: \( \text{full} \)
- \( o \): role chain (composition): \( R; S \)
- \( * \): reflexive-transitive closure: \( R^* \)
- \( id \): concept identity: \( id(C) \)
- Complex roles in number restrictions

**TBox** is internalized in extensions of ALCIO, see [76, Lemma 4.12], [54, p.3]
- Empty TBox
- Acyclic TBox (\( A \equiv C \), \( A \) is a concept name; no cycles)
- General TBox (\( C \sqsubseteq D \) for arbitrary concepts \( C \) and \( D \))

You have selected the Description Logic: SHOIN

**Complexity of reasoning problems**

<table>
<thead>
<tr>
<th>Reasoning problem</th>
<th>Complexity</th>
<th>Comments and references</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept satisfiability</td>
<td>NExpTime-complete</td>
<td>• Hardness of even ALCIO is proved in [76, Corollary 4.13]. In that paper, the result is formulated for ALCQIO, but only number restrictions of the form (( \leq 1 R )) are used in the proof.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• A different proof of the NExpTime-hardness for ALCIO is given in [54] (even with 1 nominal, and role inverses not used in number restrictions).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Upper bound for SHOIQ is proved in [77, Corollary 6.31] with numbers coded in unary (for binary coding, the upper bound remains an open problem for all logics in between ALCNTO and SHOIQ).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Important: in number restrictions, only simple roles (i.e. which are neither transitive nor have a transitive subroles) are allowed; otherwise we gain undecidability even in SHOIQ, see [46].</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Remark: recently [47] it was observed that, in many cases, one can use transitive roles in number restrictions - and still have a decidable logic! So the above notion of a simple role could be substantially extended.</td>
</tr>
<tr>
<td>ABox consistency</td>
<td>NExpTime-complete</td>
<td>By reduction to concept satisfiability problem in presence of nominals shown in [69, Theorem 3.7].</td>
</tr>
</tbody>
</table>
Complexity
Complexity Measures

• **Taxonomic** complexity
  Measured w.r.t. total size of “schema” axioms

• **Data** complexity
  Measured w.r.t. total size of “data” facts

• **Query** complexity
  Measured w.r.t. size of query

• **Combined** complexity
  Measured w.r.t. total size of KB (plus query if appropriate)
Complexity Classes

• LogSpace, PTime, NP, PSpace, ExpTime, etc
  – worst case for a given problem w.r.t. a given parameter
  – X-hard means at-least this hard (could be harder);
    in X means no harder than this (could be easier);
    X-complete means both hard and in, i.e., exactly this hard
  • e.g., SROIQ KB satisfiability is 2NExpTime-complete w.r.t.
    combined complexity and NP-hard w.r.t. data complexity

• Note that:
  – this is for the worst case, not a typical case
  – complexity of problem means we can never devise a more
    efficient (in the worst case) algorithm
  – complexity of algorithm may, however, be even higher
    (in the worst case)
DLs and Ontology Languages
DLs and Ontology Languages

- W3C's OWL 2 (like OWL, DAML+OIL & OIL) based on DL
  - OWL 2 based on SROIQ, i.e., ALC extended with transitive roles, a role box nominals, inverse roles and qualified number restrictions
    - OWL 2 EL based on EL
    - OWL 2 QL based on DL-Lite
    - OWL 2 EL based on DLP
  - OWL was based on SHOIN
    - only simple role hierarchy, and unqualified NRs
## Class/Concept Constructors

<table>
<thead>
<tr>
<th>OWL Constructor</th>
<th>DL Syntax</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>intersectionOf</td>
<td>$C_1 \sqcap \ldots \sqcap C_n$</td>
<td>Human $\sqcap$ Male</td>
</tr>
<tr>
<td>unionOf</td>
<td>$C_1 \sqcup \ldots \sqcup C_n$</td>
<td>Doctor $\sqcup$ Lawyer</td>
</tr>
<tr>
<td>complementOf</td>
<td>$\neg C$</td>
<td>$\neg$ Male</td>
</tr>
<tr>
<td>oneOf</td>
<td>${x_1} \sqcup \ldots \sqcup {x_n}$</td>
<td>{john} $\sqcup$ {mary}</td>
</tr>
<tr>
<td>allValuesFrom</td>
<td>$\forall P.C$</td>
<td>$\forall$ hasChild.Doctor</td>
</tr>
<tr>
<td>someValuesFrom</td>
<td>$\exists P.C$</td>
<td>$\exists$ hasChild.Lawyer</td>
</tr>
<tr>
<td>maxCardinality</td>
<td>$\leq nP$</td>
<td>$\leq 1$ hasChild</td>
</tr>
<tr>
<td>minCardinality</td>
<td>$\geq nP$</td>
<td>$\geq 2$ hasChild</td>
</tr>
</tbody>
</table>
## Ontology Axioms

<table>
<thead>
<tr>
<th>OWL Syntax</th>
<th>DL Syntax</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>subClassOf</td>
<td>$C_1 \sqsubseteq C_2$</td>
<td>Human $\sqsubseteq$ Animal $\sqcap$ Biped</td>
</tr>
<tr>
<td>equivalentClass</td>
<td>$C_1 \equiv C_2$</td>
<td>Man $\equiv$ Human $\sqcap$ Male</td>
</tr>
<tr>
<td>subPropertyOf</td>
<td>$P_1 \sqsubseteq P_2$</td>
<td>hasDaughter $\sqsubseteq$ hasChild</td>
</tr>
<tr>
<td>equivalentProperty</td>
<td>$P_1 \equiv P_2$</td>
<td>cost $\equiv$ price</td>
</tr>
<tr>
<td>transitiveProperty</td>
<td>$P^+ \sqsubseteq P$</td>
<td>ancestor$^+ \sqsubseteq$ ancestor</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OWL Syntax</th>
<th>DL Syntax</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>type</td>
<td>$a : C'$</td>
<td>John : Happy-Father</td>
</tr>
<tr>
<td>property</td>
<td>$\langle a, b \rangle : R$</td>
<td>$\langle$John, Mary$\rangle :$ has-child</td>
</tr>
</tbody>
</table>

- An **Ontology** is *usually* considered to be a TBox
  - but an **OWL** ontology is a mixed set of TBox and ABox axioms
Other OWL Features

• XSD datatypes and (in OWL 2) facets, e.g.,
  – integer, string and (in OWL 2) real, float, decimal, datetime, …
  – minExclusive, maxExclusive, length, …
  – PropertyAssertion( hasAge Meg "17"^^xsd:integer )
  – DatatypeRestriction( xsd:integer xsd:minInclusive "5"^^xsd:integer xsd:maxExclusive "10"^^xsd:integer )

These are equivalent to (a limited form of) DL concrete domains

• Keys
  – E.g., HasKey(Vehicle Country LicensePlate)
    • Country + License Plate is a unique identifier for vehicles

This is equivalent to (a limited form of) DL safe rules
OWL RDF/XML Exchange Syntax

E.g., Person \( \sqcap \forall \text{hasChild.}(\text{Doctor} \sqcup \exists \text{hasChild.Doctor})\):

```xml
<owl:Class>
  <owl:intersectionOf rdf:parseType=" collection">
    <owl:Class rdf:about="#Person"/>
    <owl:Restriction>
      <owl:onProperty rdf:resource="#hasChild"/>
      <owl:allValuesFrom>
        <owl:unionOf rdf:parseType=" collection">
          <owl:Class rdf:about="#Doctor"/>
          <owl:Restriction>
            <owl:onProperty rdf:resource="#hasChild"/>
            <owl:someValuesFrom rdf:resource="#Doctor"/>
          </owl:Restriction>
        </owl:unionOf>
      </owl:allValuesFrom>
    </owl:Restriction>
  </owl:intersectionOf>
</owl:Class>
```
Complexity/Scalability

• From the complexity navigator we can see that:
  – OWL (aka \textit{SHOIN}) is $\text{NExpTime}$-complete
  – OWL Lite (aka \textit{SHIF}) is $\text{ExpTime}$-complete (Oops!)
  – OWL 2 (aka \textit{SROIQ}) is $\text{2NExpTime}$-complete
  – OWL 2 EL (aka \textit{EL}) is $\text{PTIME}$-complete (Robustly scalable)
  – OWL 2 RL (aka \textit{DLP}) is $\text{PTIME}$-complete (Robustly scalable)
    • And implementable using rule based technologies
      e.g., rule-extended DBs
  – OWL 2 QL (aka DL-Lite) is in \textit{AC$^0$} w.r.t. size of data
    • same as DB query answering -- nice!
Why (Description) Logic?

- OWL exploits results of 20+ years of DL research
  - Well defined (model theoretic) **semantics**

<table>
<thead>
<tr>
<th>Constructor</th>
<th>DL Syntax</th>
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<th>FOL Syntax</th>
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<tbody>
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<td>intersectionOf</td>
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<td>$x = x_1 \lor \ldots \lor x = x_n$</td>
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<td>$\geq n P$</td>
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</table>
Why (Description) Logic?

• OWL exploits results of 20+ years of DL research
  – Well defined (model theoretic) **semantics**
  – **Formal properties** well understood (complexity, decidability)

I can’t find an efficient algorithm, but neither can all these famous people.

Why (Description) Logic?

- OWL exploits results of 20+ years of DL research
  - Well defined (model theoretic) **semantics**
  - **Formal properties** well understood (complexity, decidability)
  - Known **reasoning algorithms**

| □-rule | if 1. \((C_1 \sqcap C_2) \in \mathcal{L}(v)\), \(v\) is not indirectly blocked, and  
|        | 2. \(\{C_1, C_2\} \notin \mathcal{L}(v)\)  
|        | then \(\mathcal{L}(v) \rightarrow \mathcal{L}(v) \cup \{C_1, C_2\}\). |
| □-rule | if 1. \((C_1 \sqcup C_2) \in \mathcal{L}(v)\), \(v\) is not indirectly blocked, and  
|        | 2. \(\{C_1, C_2\} \cap \mathcal{L}(v) = \emptyset\)  
|        | then \(\mathcal{L}(v) \rightarrow \mathcal{L}(v) \cup \{E\}\) for some \(E \in \{C_1, C_2\}\). |
| ⊤-rule | if 1. \(\exists r. C \in \mathcal{L}(v_1)\), \(v_1\) is not blocked, and  
|        | 2. \(v_1\) has no safe \(r\)-neighbour \(v_2\) with \(C \in \mathcal{L}(v_1)\),  
|        | then create a new node \(v_2\) and an edge \((v_1, v_2)\)  
|        | with \(\mathcal{L}(v_2) = \{C\}\) and \(\mathcal{L}(\langle v_1, v_2 \rangle) = \{r\}\). |
| ∀-rule | if 1. \(\forall r. C \in \mathcal{L}(v_1)\), \(v_1\) is not indirectly blocked, and  
|        | 2. there is an \(r\)-neighbour \(v_2\) of \(v_1\) with \(C \notin \mathcal{L}(v_2)\)  
|        | then \(\mathcal{L}(v_2) \rightarrow \mathcal{L}(v_2) \cup \{C\}\). |
| ∀⁺-rule | if 1. \(\forall r. C \in \mathcal{L}(v_1)\), \(v_1\) is not indirectly blocked, and  
|        | 2. there is some role \(r'\) with \(\text{Trans}(r')\) and \(r' \ni r\)  
|        | 3. there is an \(r'\)-neighbour \(v_2\) of \(v_1\) with \(\forall r'. C \notin \mathcal{L}(v_2)\)  
|        | then \(\mathcal{L}(v_2) \rightarrow \mathcal{L}(v_2) \cup \{\forall r'. C\}\). |
| choose-rule | if 1. \(\leq n \forall r. C \in \mathcal{L}(v_1)\), \(v_1\) is not indirectly blocked, and  
|           | 2. there is an \(r\)-neighbour \(v_2\) of \(v_1\) with \(\{C, \neg C\} \cap \mathcal{L}(v_2) = \emptyset\)  
|           | then \(\mathcal{L}(v_2) \rightarrow \mathcal{L}(v_2) \cup \{E\}\) for some \(E \in \{C, \neg C\}\). |
| ⊳-rule | if 1. \(\geq n \forall r. C \in \mathcal{L}(v)\), \(v\) is not blocked, and  
|        | 2. there are not \(n\) safe \(r\)-neighbours \(v_1, \ldots, v_n\) of \(v\)  
|        | with \(C \in \mathcal{L}(v_i)\) and \(v_i \neq v_j\) for \(1 \leq i < j \leq n\). |
Why (Description) Logic?

• OWL exploits results of 20+ years of DL research
  – Well defined (model theoretic) semantics
  – **Formal properties** well understood (complexity, decidability)
  – Known **reasoning algorithms**
  – **Scalability** demonstrated by *implemented systems*
Tools, Tools, Tools

**Major benefit** of OWL has been huge increase in range and sophistication of tools and infrastructure:
**Tools, Tools, Tools**

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- Editors/development environments
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- Editors/development environments
- Reasoners

![Hermit](Hermit.png)  ![FaCT++] (FaCT++).png

![Racer](Racer.png)  ![Pellet](Pellet.png)

![KAON2](KAON2.png)  ![CEL](CEL.png)
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**Major benefit** of OWL has been huge increase in range and sophistication of tools and infrastructure:

- Editors/development environments
- Reasoners
- Explanation, justification and pinpointing
- Integration and modularisation
- APIs, in particular the **OWL API**
Ontology -v- Database
Obvious Database Analogy

- Ontology axioms analogous to DB schema
  - Schema describes structure of and constraints on data
- Ontology facts analogous to DB data
  - Instantiates schema
  - Consistent with schema constraints
- But there are also important differences…
### Obvious Database Analogy

**Database:**

- **Closed world assumption (CWA)**
  - Missing information treated as false
- **Unique name assumption (UNA)**
  - Each individual has a single, unique name
- **Schema behaves as constraints on structure of data**
  - Define legal database states
- **Single canonical model**
  - Can check entailments (query answers) w.r.t. this model

**Ontology:**

- **Open world assumption (OWA)**
  - Missing information treated as unknown
- **No UNA**
  - Individuals may have more than one name
- **Ontology axioms behave like implications (inference rules)**
  - Entail implicit information
- **Typically multiple models**
  - Need to check entailment w.r.t. all models
Database -v- Ontology

E.g., given the following **ontology/schema**:

- \( \text{HogwartsStudent} \equiv \text{Student} \land \exists \text{attendsSchool} \cdot \text{Hogwarts} \)
- \( \text{HogwartsStudent} \sqsubseteq \forall \text{hasPet} \cdot (\text{Owl or Cat or Toad}) \)
- \( \text{hasPet} \equiv \text{isPetOf}^- \) (i.e., hasPet inverse of isPetOf)
- \( \exists \text{hasPet}. \top \sqsubseteq \text{Human} \) (i.e., domain of hasPet is Human)
- \( \text{Phoenix} \sqsubseteq \forall \text{isPetOf} \cdot \text{Wizard} \) (i.e., only Wizards have Phoenix pets)
- \( \text{Muggle} \sqsubseteq \neg \text{Wizard} \) (i.e., Muggles and Wizards are disjoint)
Database -v- Ontology

And the following facts/data:

HarryPotter: Wizard
DracoMalfoy: Wizard
HarryPotter hasFriend RonWeasley
HarryPotter hasFriend HermioneGranger
HarryPotter hasPet Hedwig

Query: Is Draco Malfoy a friend of HarryPotter?

- DB: No
- Ontology: Don’t Know
  OWA (didn’t say Draco was not Harry’s friend)
Database -v- Ontology

And the following facts/data:

- HarryPotter: Wizard
- DracoMalfoy: Wizard
- HarryPotter hasFriend RonWeasley
- HarryPotter hasFriend HermioneGranger
- HarryPotter hasPet Hedwig

Query: How many friends does Harry Potter have?

- DB: 2
- Ontology: at least 1

No UNA (Ron and Hermione may be 2 names for same person)
Database -v- Ontology

And the following facts/data:

HarryPotter: Wizard
DracoMalfoy: Wizard
HarryPotter hasFriend RonWeasley
HarryPotter hasFriend HermioneGranger
HarryPotter hasPet Hedwig

RonWeasley ≠ HermioneGranger

Query: How many friends does Harry Potter have?

- DB: 2
- Ontology: at least 2

OWA (Harry may have more friends we didn’t mention yet)
Database -v- Ontology

And the following facts/data:

HarryPotter: Wizard
DracoMalfoy: Wizard
HarryPotter hasFriend RonWeasley
HarryPotter hasFriend HermioneGranger
HarryPotter hasPet Hedwig

RonWeasley \neq HermioneGranger

\[ \text{HarryPotter: } \forall \text{hasFriend.} \{\text{RonWeasley}\} \uplus \{\text{HermioneGranger}\} \]

Query: How many friends does Harry Potter have?

- DB: 2
- Ontology: 2!
Database -v- Ontology

**Inserting** new facts/data:

Fawkes: Phoenix
Fawkes isPetOf Dumbledore

What is the response from DBMS?

- Update rejected: *constraint violation*
  
  Domain of hasPet is Human; Dumbledore is not Human (CWA)

What is the response from Ontology reasoner?

- **Infer** that Dumbledore is Human (domain restriction)
- Also infer that Dumbledore is a Wizard (only a Wizard can have a phoenix as a pet)
DB Query Answering

• Schema plays no role
  – Data must explicitly satisfy schema constraints

• Query answering amounts to model checking
  – I.e., a “look-up” against the data

• Can be very efficiently implemented
  – Worst case complexity is low (logspace) w.r.t. size of data
Ontology Query Answering

- Ontology axioms play a powerful and crucial role
  - Answer may include implicitly derived facts
  - Can answer conceptual as well as extensional queries
    - E.g., Can a Muggle have a Phoenix for a pet?

- Query answering amounts to theorem proving
  - I.e., logical entailment

- May have very high worst case complexity
  - E.g., for OWL, NP-hard w.r.t. size of data
    (upper bound is an open problem)
  - Implementations may still behave well in typical cases
  - Fragments/profiles may have much better complexity
Ontology Based Information Systems

• Analogous to relational database management systems
  – Ontology \( \approx \) schema; instances \( \approx \) data

• Some important (dis)advantages
  + (Relatively) easy to maintain and update schema
    • Schema plus data are integrated in a logical theory
  + Query answers reflect both schema and data
  + Can deal with incomplete information
  + Able to answer both intensional and extensional queries
  – Semantics can seem counter-intuitive, particularly w.r.t. data
    • Open -v- closed world; axioms -v- constraints
  – Query answering (logical entailment) may be much more difficult
    • Can lead to scalability problems with expressive logics
Ontology Based Information Systems

- Analogous to relational database management systems
  - Ontology ≈ schema
- Some important (+) advantages
  + (Relatively) easy to maintain and update schema
  + Schema plus data are integrated in a logical theory
  + Query answers reflect both schema and data
  + Can deal with incomplete information
  + Able to answer both intensional and extensional queries
- Semantics can seem counter-intuitive, particularly w.r.t. data
  - Open-versus-closed world; axioms-versus-constraints
- Query answering (logical entailment) may be much more difficult
  - Can lead to scalability problems with expressive logics